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Second-Order Five-Dimensional Chern–Simons Gravity

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Continuing our previous discussion of the canonical covariant formalism (Zandron, O. S. (in press). *International Journal of Theoretical Physics*), the second-order canonical fünfbein formalism of the topological five-dimensional Chern–Simons gravity is constructed. Since this gravity model naturally contains a Gauss–Bonnet term quadratic in curvature, the second-order formalism requires the implementation of the Ostrogradski transformation in order to introduce canonical momenta. This is due to the presence of second time-derivatives of the fünfbein field. By performing the space–time decomposition of the manifold M^5, the set of first-class constraints that determines all the Hamiltonian gauge symmetries can be found. The total Hamiltonian as generator of time evolution is constructed, and the apparent gauge degrees of freedom are unambiguously removed, leaving only the physical ones.

KEY WORDS: canonical covariant formalism; Chern–Simons gravity.

1. INTRODUCTION

Recently, the five-dimensional Chern–Simons gravity theory was formulated in the framework of the canonical covariant formalism (CCF) (Zandron, in press). In this context the primary constraints were found, and the total Hamiltonian as a first-class dynamical quantity strongly conserved was studied. Next the toroidal dimensional reduction of the model was carried out by assuming that the vacuum topology is given by $M^4 \times S^1$. So, the effective interactions between the gravitational field and the electromagnetic one can be studied. Moreover it was shown how the Gauss–Bonnet term appearing in the original five-dimensional model gives rise to all the possible nonlinear corrections to electromagnetism and the nonminimal coupling to gravity.

In spite of the fact that the CCF is not a proper Hamiltonian formalism because it does not really propagate data defined on an initial hypersurface Σ, it allows one to study many essential properties of the classical canonical formalism

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for constrained systems. This is carried out in a more simple way than provided by the usual canonical fünfbein formalism (CFF). However, when the model is considered from the quantum point of view, the CFF and therefore the use of the Poisson brackets is needed.

In Zandron (in press) it was also shown that the relation between the CCF and the usual CFF is not trivial. The integral relationship relating the form brackets introduced in the CCF with the standard Poisson brackets of the CFF was analyzed. This was done in the first-order CCF, i.e., when both field variables, the fünfbein $\tilde{V}^A$ and the spin connection $\tilde{\omega}^{AB}$ are considered as independent variables, so only first-time derivatives appear in the formalism.

The Riemannian case $\tilde{R}^A = d\tilde{V}^A - \tilde{\omega}^A_{.B} \wedge \tilde{V}^B = 0$ was assumed in that paper. When the torsion equation of motion is taken as an strongly equal to zero constraint, it allows to solve for the spin connection $\tilde{\omega}^A_{.B} = \tilde{\omega}^A_{.B}(\tilde{V})$ in terms of the fünfbein obtaining the second-order formalism. Therefore the Hamiltonian formalism is completed when the second-order CFF is implemented. In this context the set of first-class constraints which determines all the Hamiltonian gauge symmetries must be constructed. So, the apparent gauge degrees of freedom can be unambiguously removed leaving only the physical ones. Again, the knowledge of the first-class constraints that verify the constraint algebra is essential from the quantum point of view.

As known, because of the presence of the Gauss–Bonnet term the five-dimensional Chern–Simons gravity model is a higher-derivative theory. It contains second time-derivatives on the fünfbein that cannot be eliminated by partial integration. So, it is not possible go over to the second-order formalism directly from the CCF. Precisely, the higher-derivative character of the theory is made evident in the second-order formalism.

Consequently, in the framework of the Dirac formalism we are in presence of a constrained Hamiltonian system with a singular higher-order Lagrangian. Therefore, in order to introduce canonical momenta in this higher derivative model, the Ostrogradski transformation must be considered (Kentwell, 1988; Kersten, 1988; Nesterenko, 1989; Nesterenko and Nguyen, 1988; Zi-ping, 1990, 1991a,b).

On the other hand, the primary constraints we have found in the framework of the CCF (see Eqs. (3.5) and (3.7) of Zandron (in press)), will be no longer a relationship between field and momentum. These relations depending on the velocities cease to be constraints, and so new constraints must be expected.

The aim of the present paper is to construct the second-order Hamiltonian formalism for the topological five-dimensional Chern–Simons gravity theory. This is done by performing the space–time decomposition in M^5, extending the results given in Nelson and Regge (1986).

This paper is organized as follows: In section 2, the space–time decomposition in the five-dimensional manifolds M^5 is carried out. In section 3, the torsion two-form $R^a = 0$ equation is used as a strongly equal to zero constraint and the metricity condition is introduced to determine the spin connection. In section 4, the model is arranged in order to derive the second-order hamiltonian formalism (Dirac, 1962; Foussats *et al.*, 1992; Nelson and Teitelboim, 1977; 1978). After introducing the Ostrogradski transformation, the new first-class constraints are studied. Finally, the total Hamiltonian generator of time evolutions is written.

2. THE SPACE–TIME DECOMPOSITION IN THE FIVE-DIMENSIONAL MANIFOLDS M^5

With the purpose to construct the second-order Hamiltonian formalism for the model, the first step is to carry out the space–time decomposition in M^5. The procedure is similar to that given in Nelson and Regge (1986) for M^4, where all the definition must be extended to five dimensions. Next, all the equations and quantities given in form language must be written in components. The dynamical fields must be considered only as rcduccd forms, i.e., forms defined on the physical space given by the coset manifold $M^5 = G/H$, where G is the group manifold and the bosonic group $H \subset G$ is the exact gauge symmetry group (Zandron, in press). Moreover, we assume that the reduced forms defined on M^5 are written in the holonomic basis dx^μ. Therefore, equations, fields, and forms must be projected on a space-like $x^0 = t = t^0$ hypersurface Σ of four dimensions. This is done by considering the injection map $\chi : \Sigma \to M^5$ in such a way that the associated pull-back χ^* acts on any generic form by setting $t = t^0$ and $dt = 0$.

From now on we use Greek indices $\mu, \nu, \rho, \ldots = 0, 1, 2, 3, 4$, for space–time tensors (world indices); Latin indices $a, b, c, \ldots$ for tangent space (Lorentz indices); and Latin indices $i, j, k, \ldots$ for label spatial components only.

Moreover, in the (CFF) the space–time split of the fünfbein $V^{(5)a} = V_\mu^{(5)a}\, dx^\mu$, and the five-dimensional metric tensor $g_{\mu\nu}^{(5)}$ must be considered. The fünfbein is split according to

$$V_{ai}^{(5)} = V_{ai}^{(4)} = V_{ai}, \tag{2.1a}$$

$$V_a^{(4)i} = V_a^i, \tag{2.1b}$$

$$V_a^{(5)i} = V_a^{(4)i} + (N^\perp)^{-1} N^i n_a, \tag{2.1c}$$

$$V_a^i V_{bi} = \eta_{ab} + n_a n_b, \tag{2.1d}$$

where the normal $n_a = n^\mu V_{a\mu}$ to the hypersurface Σ satisfies

$$n_a = -N^\perp V_a^{(5)0}, \tag{2.2a}$$

$$n_a V_i^a = 0, \tag{2.2b}$$

$$n_a n^a = -1, \tag{2.2c}$$

and $n_\mu = (-N^\perp, 0, 0, 0, 0)$. In the above equation N^i and $N^\perp$ are respectively the usual shift and lapse functions which determine the components of the five-dimensional metric tensor $g^{(5)}_{\mu\nu}$. The five-dimensional metric tensor $g^{(5)}_{\mu\nu}$ split according to $N^{\perp} = (-g^{00})^{1/2}$, $N_i = g_{0i}$, $g = \det(g^{(4)})$, $(-g^{(5)})^{1/2} = N^{\perp} g^{1/2}$.

An arbitrary vector $\mathcal{V}^{(5)a}$ can be decomposed as follows

$$\mathcal{V}^{(5)a} = \mathcal{V}^{\perp} n^a + \mathcal{V}^i V_i^a, \tag{2.3a}$$

where

$$\mathcal{V}^{\perp} = -\mathcal{V}_{\perp} = -n_a \mathcal{V}^a, \qquad \mathcal{V}_i = \mathcal{V}^{(5)a} V_{ai}. \tag{2.3b}$$

The alternating tensors $\varepsilon_{i_1 \ldots i_4}$ on the hypersurface Σ and $\varepsilon_{0, i_1 \ldots i_4}$ on the manifold M^5 are related by the equation: $N^{\perp} \varepsilon_{i_1 \ldots i_4} = -\varepsilon_{0, i_1 \ldots i_4}$. Moreover, $\varepsilon_{\mu_1 \ldots \mu_5} = V^{(5)}_{a_1 \mu_1} \ldots V^{(5)}_{a_5 \mu_5} \varepsilon^{a_1 \ldots a_5}$.

The spatial components of the two one-form gauge fields in the dx^i, basis on Σ are called respectively: $-\omega_i^{(5)ab}(x)$ and $V_i^a(x)$.

The corresponding set of canonical momenta π_{ab} and π_a can be also written in the basis. We define the relationship between such momenta and their spatial components through the general equation

$$\pi_{\Sigma} = \frac{1}{(D - 1 - n)!} \pi_{\Sigma}^{i_1 \ldots i_n}(x) \varepsilon_{i_1 \ldots i_n j_1 \ldots j_m} dx^{j_1 \ldots j_m} g^{-1/2} \tag{2.4}$$

where $n + m = D - 1 = 4$ and being n the form degree of the field μ^{Σ} canonical conjugate to the momentum π_{Σ}.

For the constraints Φ_{ab} and Φ_a associated to the canonical momenta, similar expressions to that given in (2.4) are hold.

Now the following Poisson brackets between the components of the field and canonical conjugate momenta can be written

$$\left[\omega_i^{ab}(x), \pi^i_{cd}(y)\right] = -\left[\pi^j_{cd}(y), \omega_i^{ab}(x)\right] = \delta^{ab}_{[cd]} \delta_i^j \delta^4(x, y), \tag{2.5a}$$

$$\left[V^a_{.i}(x), \pi_b^j(y)\right] = -\left[\pi_b^j(y), V^a_{.i}(x)\right] = \delta^a_b \, \delta_i^j \, \delta^4(x, y), \tag{2.5b}$$

Heretofore, in Zandron (in press) the spin connection and the fünfbein were taken as independent dymamical field working within the first-order CFF. It does not allow the identification of the true dynamical fields by removing the gauge degrees of freedom from the physical ones. Therefore the second-order formalism must be developed.

3. TORSION EQUATION AND METRICITY CONDITION

In these models the torsion equation plays an important role because it allows the construction of the second-order canonical formalism starting from the first-order one by solving certain field equations considered as constraints on curvatures (Macías and Lozano, 2001). In the Riemannian case the torsion two-form $R^a = 0$ equation must be considered as an strongly equal to zero constraint.

So, the higher-derivative character of the theory is made evident in the framework of the second-order formalism. From the torsion equation it is easy to obtain the solution for the spin connection one-form $\omega^{(5)ab}$, whose components in the holonomic basis have the well-known expression for pure gravity in five dimensions, i.e.,

$$\omega_\mu^{(5)ab}(V) = \frac{1}{2}V^{(5)a\nu}\left(\partial_\mu V_\nu^{(5)b} - \partial_\nu V_\mu^{(5)b}\right) - \frac{1}{2}V^{(5)b\nu}\left(\partial_\mu V_\nu^{(5)a} - \partial_\nu V_\mu^{(5)a}\right)$$
$$-\frac{1}{2}V^{(5)a\rho}V^{(5)b\sigma}\left(\partial_\rho V_{c\sigma}^{(5)} - \partial_\rho V_{c\rho}^{(5)}\right)V_\mu^{(5)c}. \tag{3.1a}$$

that also can be written

$$\omega_\mu^{(5)ab}(V) = \frac{1}{2}\left(\Theta_{\mu\nu\rho}^{(5)} - \Theta_{\nu\rho\mu}^{(5)} + \Theta_{\rho\mu\nu}^{(5)}\right)V^{(5)a\nu}V^{(5)b\rho} \tag{3.1b}$$

where

$$\Theta_{\mu\nu\rho}^{(5)} = \left(\partial_\mu V_\nu^{(5)a} - \partial_\nu V_\mu^{(5)a}\right)V_{a\rho}^{(5)} \tag{3.2}$$

On the other hand, in the second-order CFF by considering the fünfbein postulate on both the manifold M^5 and the four-dimensional hypersurface Σ, the spin connections $\omega_\mu^{(5)ab}$ and $\omega_i^{(4)ab}$ can be determined completely.

The metricity condition or fünfbein postulate implicates that "the fünfbein is covariantly constant." That is the full covariant derivative including both the spin and the world (metric) connection satisfies the standard metricity condition

$$\partial_\mu V_{.\nu}^{(5)a} + \omega_\mu^{(5)ab}V_{b\nu}^{(5)} - \Gamma_{.\mu\nu}^{(5)\rho}V_{.\rho}^{(5)a} = 0. \tag{3.3}$$

where $\Gamma_{.\mu\nu}^{(5)\rho}$ is the affine connection on the five-dimensional manifold M^5.

By considering the spin connection $\omega^{(4)ab}$ and the affine connection $\Gamma_{.ij}^{(4)i}$ on the hypersurface Σ, we also have

$$\partial_k V_{.j}^a + \omega_k^{(4)ab}V_{bj} - \Gamma_{.kj}^{(4)i}V_{.i}^a = 0, \tag{3.4}$$

by virtue of the metricity condition on the four-dimensional hypersurface Σ. Also, multiplying the Eq. (3.4) by n_a holds

$$\partial_k n^a + \omega_k^{(4)ab}n_b = 0. \tag{3.5}$$

Therefore the Eqs. (3.3) and (3.4) determine completely both spin connections $\omega^{(5)ab}$ and $\omega^{(4)ab}$.

After some algebraic manipulations the well-known relationship between the spatial components of both spin connections can be found

$$\omega_i^{(5)ab} = \omega_i^{(4)ab} + (n^b V^{aj} - n^a V^{bj})K_{ij}, \tag{3.6}$$

where K_{ij} is the extrinsic curvature on the four-dimensional surface Σ in the manifold M^5. The extrinsic curvature tensor K_{ij} is defined by the following general equation

$$K_{ij} = \frac{1}{N^{\perp}}(-\dot{g}_{ij} + N_{i\|j} + N_{j\|i}), \tag{3.7}$$

where the double stroke $\|$ denotes the covariaut derivative on the four-surface Σ only including the affine connection.

Moreover, for the component $\omega_{\perp}^{\cdot ab}$ the following equation holds

$$N^{\perp}\omega_{\perp}^{ab} = \frac{1}{2}\big(V^{ak}\delta_N V_k^b - n^a \delta_N n^b - [a \to b]\big) - (V^{ak}n^b - V^{bk}n^a)\partial_k N^{\perp} \tag{3.8}$$

where

$$\delta_N\, V^{ak} = \partial_0\, V^{ak} - \mathcal{L}_{N^k}\, V^{ak}, \tag{3.9a}$$

$$\delta_N\, n^a = \partial_0\, n^a - \mathcal{L}_{N^k}\, n^a, \tag{3.9b}$$

and $\mathcal{L}_{N^k}$ stands for the Lie derivative operator along N^k in the four-dimensional hypersurface Σ.

As it can be seen by means of lengthy but direct calculations, both the constraints and the canonical Hamiltonian $\mathcal{H}_{\text{can}}$ can be written in terms of the canonical momenta and the quantities V_{ai}, $N^{\perp}$, N_i, $\omega_i^{(4)ab}$, $\omega_{\perp}^{\cdot ab}$, and K_{ij}.

4. SECOND-ORDER HAMILTONIAN FORMALISM AND NEW CONSTRAINTS

As commented above, the higher-derivative character of this model does not allow go over to the second-order formalism directly from the CCF. This is due to the presence of second time-derivatives when the spin-connection is written in terms of the fünfbein according to (3.1).

Consequently, we must turn to the initial five-form Lagrangian density (Macías and Lozano, 2001)

$$\mathcal{L} = \varepsilon_{abcde}\left(R^{bc}\wedge R^{de}\wedge V^a + \frac{2}{3}\lambda\, R^{de}\wedge V^a \wedge V^b \wedge V^c + \frac{1}{5}\lambda^2 V^a\wedge V^b\wedge V^c\wedge V^d\wedge V^e\right), \tag{4.1}$$

where V^a and R^{bc} are respectively the fünfbein one-form and the Riemannian curvature two-form in the manifold M^5.

Now, the Lagrangian density (4.1) must be written in components. Once the torsion equation is taken into account, and without considering total exterior derivatives, the Lagrangian reads

$$\mathcal{L} = -N^{\perp}g^{1/2}\varepsilon_{abcde}\varepsilon^{\alpha\mu\nu\rho\tau}\big[\partial_\alpha\omega_\mu^{(5)de}\big(\omega_\nu^{(5)bc}\omega_\rho^{(5)af}V_{\tau f}^{(5)} + 2\omega_\nu^{(5)bf}\omega_\rho^{(5)fc}V_{\tau a}^{(5)}\big) - \omega_\alpha^{(5)bf}\omega_\mu^{(5)fc}\omega_\nu^{(5)dg}\omega_\rho^{(5)ge}V_\tau^{(5)a} - 2\,\lambda\,\omega_\alpha^{(5)de}\omega_\mu^{(5)af}V_\nu^{(5)f}V_\rho^{(5)b}V_\tau^{(5)c}\big], \tag{4.2}$$

being the last term of (4.1) a constant one.

The first two terms in (4.2) originate the second time-derivative terms on the fünfbein, when the Eq. (3.1) is used.

In the Chern–Simons expression it is not possible to eliminate the higher-derivative terms by partial integration. Consequently, in the framework of the Dirac formalism we are in the presence of a constrained Hamiltonian system with a singular higher-order Lagrangian.

Therefore, at this stage it is necessary to consider the Ostrogradski transformation (Nesterenko, 1989; Zi-ping, 1990, 1991a,b). Following the steps given in Nesterenko (1989) and Zi-ping (1990, 1991a,b), canonical momenta in this higher-derivative theory must be introduced.

The space–time decomposition we use is that given in (2.1). We start by defining the following independent dynamical field variables

$$V^{(5)}_{a\mu} = \left(V^{(5)}_{ai}; V^{(5)}_{a0} = n_a N^{\perp} + N^i V_{ai}\right) \tag{4.3a}$$

$$B_{a\mu} = \partial_0 V^{(5)}_{a\mu} \tag{4.3b}$$

The Ostrogradski transformation introduces respectively the following canonical momenta

$$\Pi^{(1)\mu}_a = \frac{\partial \mathcal{L}}{\partial B^a_\mu} - \partial_\nu \left[\frac{\partial \mathcal{L}}{\partial\left(\partial_\nu B^a_\mu\right)}\right] \tag{4.4a}$$

$$\Pi^{(2)\mu}_a = \frac{\partial \mathcal{L}}{\partial\left(\partial_0 B^a_\mu\right)} \tag{4.4b}$$

where the Poisson brackets for canonical conjugate variables are given by

$$\left[V^{(5)a}_\nu(x), \Pi^{(1)\mu}_b(y)\right] = -\left[\Pi^{(1)\mu}_b(y), V^{(5)a}_\nu(x)\right] = \delta^a_b\, \delta^\mu_\nu\, \delta(x-y), \tag{4.5a}$$

$$\left[B^a_\nu(x), \Pi^{(2)\mu}_b(y)\right] = -\left[\Pi^{(2)\mu}_b(y), B^a_\nu(x)\right] = \delta^a_b\, \delta^\mu_\nu\, \delta(x-y). \tag{4.5b}$$

At this stage by using the space–time decomposition, also the Eq. (4.2) is written in terms of the quantities V_{ai}, $N^{\perp}$, N_i, $\omega^{(4)ab}_i$, $\omega^{\cdot ab}_{\perp}$, and K_{ij}. From the Eq. (4.4) and by means of straightforward but heavy algebraic manipulation the different momenta can be computed and the following results are found:

1. The relationships between fields and canonical conjugate momenta independent on the velocities give rise to the following primary constraints

$$\phi^{(2)0}_c = \Pi^{(2)0}_c \approx 0, \tag{4.6a}$$

$$\phi^{(2)i}_c = \Pi^{(2)i}_c - f^i_c\left(\omega^{\cdot ab}_k, V^a_k\right) \approx 0, \tag{4.6b}$$

   where $f^i_c(\omega^{\cdot ab}_k, V^a_k)$ is a functional only of the spatial components and the perpendicular component of the spin connection, and the spatial components of the fünfbein,

$$\phi^{(1)0}_c = \Pi^{(1)0}_c - g^0_c\left(\omega^{\cdot ab}_k\ V^a_k\right) \approx 0. \tag{4.6c}$$

Analogously, the functional g is a cumbersome expression depending on the spatial components and the perpendicular component of the spin connection, its spatial derivatives, and the components of the fünfbein. [We do not write here the explicit expressions of the functional f and g because this is not necessary for our purpose.]

2. The spatial components of the momentum (4.4a), i.e., $\Pi_c^{(1)i}$ is a cumbersome expression depending on the velocities.

By means of these momenta, the canonical Hamiltonian $\mathcal{H}_{\text{can}}$ remains defined by

$$\mathcal{H}_{\text{can}} = B_\mu^a \Pi_a^{(1)\mu} + \dot{B}_\mu^a \Pi_a^{(2)\mu} - \mathcal{L} \tag{4.7}$$

where it was replaced by $\partial_0 V_\mu^a p$ or B_μ^a. We note that the canonical Hamiltonian is formed by eliminating only the velocities $\partial_0 B_\mu^a$. The field B_μ^a cannot be eliminated from the formalism when we treat with higher-derivative Lagrangians (Nesterenko, 1989). Once the explicit expression of the $\mathcal{L}$ is used in (4.7) the velocities $\dot{B}_\mu^a$ are eliminated.

Finally, the total Hamiltonian generator of time evolution of generic functionals is given by

$$H_T = \int d^4 \mathcal{H}_T, \tag{4.8}$$

where

$$\mathcal{H}_T = \mathcal{H}_{\text{can}} + \lambda_{.\mu}^{(2)c}\, \phi_c^{(2)\mu} + \lambda_{.0}^{(1)c}\, \phi_c^{(1)0}. \tag{4.9}$$

The arbitrary Lagrange multipliers are evaluated by means of the Hamilton equations $\dot{A} = [A, H_T]_{PB}$.

At this stage, from the stationary primary constraints, it is possible to define successively the secondary constraints according to the well-known Dirac algorithm

$$\Omega_c^{(k)} = \left[\Omega_c^{(k-1)}, H_T\right]_{PB}. \tag{4.10}$$

This algorithm must be continued until $\Omega_c^{(k)}$ satisfies

$$\Omega_c^{(k+1)} = \left[\Omega_c^{(k)}, H_T\right]_{PB} = C_{.cn}^a\, \Omega_a^{(n)}. \tag{4.11}$$

It can be shown that in the model under consideration there is a set of secondary constraints. By explicit computation it can be shown that

$$\Omega_c^{(1)} = \dot{\phi}_c^{(2)0} = \left[\phi_c^{(2)0}, H_T\right]_{PB} \approx 0, \tag{4.12}$$

is a weakly zero quantity.

From now on, following the Dirac's prescriptions, the procedure can be continued for each one of the constraints. The Poisson brackets different from zero which must be evaluated are essentially $[\Pi_c^{(2)i}(x), \omega_\mu^{ab}(y)]_{PB}$ and $[\Pi_c^{(1)\rho}(x), \omega_\mu^{ab}(y)]_{PB}$. Although the explicit computation is straightforward it involves heavy algebraic manipulations.

Moreover, when the computation of the Poisson brackets is carried out it can be seen that none of the secondary constraints is first-class.

Some conclusions can be obtained. Looking at the primary constraints (4.6) and taking into account the secondary constraints constructed by means of application of the Dirac algorithm (4.10), can be seen that the unique primary constraint having vanishing Poisson brackets with all the other ones is $\phi_c^{(2)0}$. So, the primary constraint $\phi_c^{(2)0}$ is first-class and corresponds to a gauge invariance of the model under a local gauge transformation. The other possible first-class constraints are constructed by considering appropriate linear combination of constraints. As well known these constraints are related with the generators M_{ab} of the local Lorentz group (Nelson and Regge, 1986).

Finally, it can be said that the five-dimensional Chern–Simons gravity theory in the second-order formalism has primary and secondary constraints. This set has constraints of first- and second-class ones. The presence of second-class constraints makes necessary to follow the prescriptions of the Dirac formalism. In this sense, the Dirac brackets must be first defined from the Poisson brackets, and next the second-class constraints must be eliminated from the formalism by taking them strongly equal to zero.

5. CONCLUSIONS

Recently (Zandron, in press), the topological five-dimensional Chern–Simons gravity was formulated in the framework of the first-order extended canonical covariant formalism (CCF). The relation between the CCF and the usual first-order canonical formalism written in components was also given. This was done by means of a nontrivial integral relationship between the form brackets and the usual Poisson brackets.

As it was shown, the CCF is not a proper canonical formalism because it does not propagate data defined on an initial hypersurface as it is required by a standard mechanical system.

In spite of this, at classical level the CCF is a powerful method to understand the structure of the gravitational field, particularly in more than four dimensions and for higher curvature gravity models (Zandron, in press). It is covariant in all their steps because of the use of exterior algebra. This allows to find the equations of motion and the constraints in a very simple way, without introducing complicate calculations.

Moreover, from the CCF only is possible go over to the proper canonical formalism in the first-order formulation, i.e., when the spin connection and the fünfbein are taken as independent field variables.

Contrarily to what happens in the CCF, in the second-order CFF after long and heavy algebraic manipulations, cumbersome noncovariant expressions for the physical quantities are obtained.

The torsion equation allows to obtain the second-order canonical formalism starting from the first-order one. In the Riemannian case the torsion two-form $R^a = 0$ equation must be considered as a strongly equal to zero constraint, and so the spin connection is solved in terms of the fünfbein.

Because of the higher-derivative character of the model made evident in the second-order formalism, the presence of second time-derivatives on the fünfbein field makes necessary the implementation of the Ostrogradski transformation in order to introduce canonical momenta. Essentially this implicate to take the first time-derivate of the fünfbein as an independent dynamical field.

Finally, by performing the space–time decomposition in M^5, and by using the Eqs. (3.1) and (3.6), the Lagrangisn density (4.2) is written in terms of the quantities V_{ai}, $N^{\perp}$, $\omega_i^{(4)ab}$, $\omega_{\perp}^{\cdot ab}$, and K_{ij} Later on, this constrained Hamiltonian system must be treated as usual according to the Dirac prescriptions.

The canonical Hamiltonian is evaluated from the Eq. (4.7). Later on, the total Hamiltonian (4.8) as generator of time evolution can be given in terms of the first-class constraints which closes the constraints algebra. Therefore, all the Hamiltonian gauge symmetries remain determined and the apparent gauge degrees of freedom csn be unambiguously removed leaving only the physical ones. This last step is indispensable when the model is considered from the quantum point of view.

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